

# Mathematical Ways of Operating: an fMRI Study with 12-year-old Participants

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## BACKGROUND

### • The Splitting Operation

From a neo-Piagetian perspective, the fundamental building blocks for modeling students' mathematics are mental actions, or operations, that have been organized within structures for reversing and composing them.

Iterating and partitioning constitute fundamental operations for modeling students' fractions schemes. Iterating involves making connected copies of a part, measuring off a length or area; partitioning involves producing equally sized parts from a continuous whole.

Students can also learn to coordinate partitioning and iterating as inverse operations so that one operation nullifies the other. By reflecting on this way of operating – sequentially coordinating partitioning and iterating – students can reorganize the two operations as a single operation, called splitting. Recent research suggests that students who can operate in this way have distinct advantages over other students in developing more advanced conceptions of fractions.

### • Units Coordination

Consider the following task: A student is given a short bar, a medium bar, and a long bar and is told to pretend that the short bar fits into the medium bar three times and that the medium bar fits into the long bar four times; then the student is asked to determine how many times the small bar would fit into the long bar.

Recent research has identified the inability to coordinate two and three levels of units as a major root cause for poor performance across the domains of whole number multiplication, fractions concepts, integer addition, and algebraic reasoning.

## RESULTS

- The variable periods between instruction offset and button response modeled as a task-related epochs for each participant
- Button responses modeled as separate events

### • Behavioral interviews

- Experimenters interacted with participants to encourage reasoning aloud and via gestures
- Participants explained reasoning using paper manipulables
- Participants classified according to levels of units coordination and as splitting or non-splitting



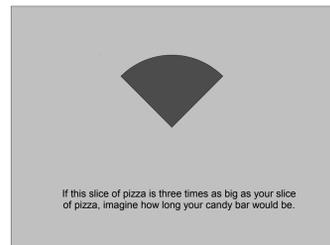
Images from video of a participant using iterating (**left**) and partitioning (**below**) in conjunction with a paper manipulable while answering questions similar to those shown at right.



## METHODS

### • Iterating, Partitioning & Splitting

- Questions designed to recruit each operation used identical images and very similar wording.
- **Iterating:**  
"If your candy bar is three times as long as this one, imagine how long would your candy bar be."
- **Partitioning:**  
"If you wanted to share this candy bar equally among five people, imagine how much each person would get."
- **Splitting:**  
"If this candy bar is six times as long as your candy bar, imagine how long your candy bar would be."

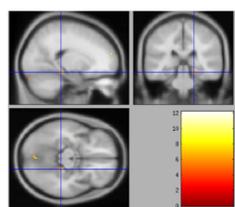
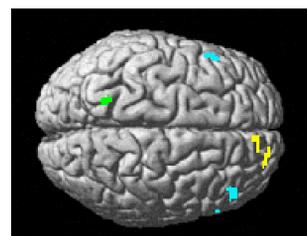
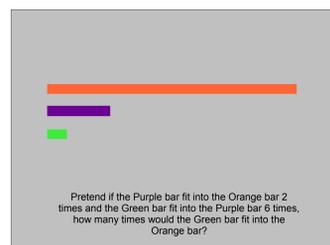


### • fMRI session

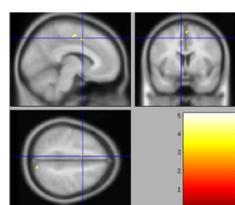
- 7 12-year-olds (1 male) participated
- All fMRI participants demonstrated splitting as a separate skill from partitioning and iterating
- Button press to indicate when answer known for iterating, partitioning and splitting
- Instructions given visually and via synthesized speech audio
- Fixation baseline

### • Units Coordination

- Three questions per display assessed 1/2/3 levels of units coordination
- fMRI participants: behavioral assessment indicated ability to coordinate up to 3 levels
- Multiple choice responses in scanner

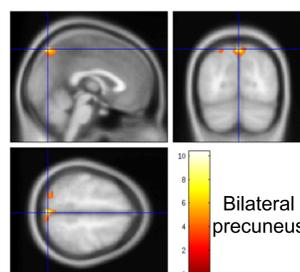


Additional iterating focus:  
Right parahippocampal gyrus



Splitting versus fixation baseline:  
Bilateral supplementary motor area (SMA);  
Right precuneus

### Units Coordination



## CONCLUSIONS

- All participants in our study were able to use the splitting operation to their advantage while judging fractions and coordinating multiple levels of units
- Among this sample of adolescents, splitting activated a distinct posterior parietal region when contrasted with iterating and partitioning
- The frontal foci seen with iterating and partitioning may be consistent with the relatively laborious and serial nature of these simpler operations

**Splitting:**  
Left SPL  
(superior parietal lobule)  
BA 7

**Partitioning:**  
Bilateral MFG  
(middle frontal gyrus)  
BA 46

**Mutually-exclusive contrasts**  
e.g. Splitting minus  
(Partitioning + Iterating)

**Iterating:**  
Right SFG  
(superior frontal gyrus)  
BA 9